Michael T. Davis Pre-Calculus	1.5 Quadratic Functions September 10-15, 2015

Name: Solutions / Answers

Key Topics to Be Covered

- Graphing quadratic functions (parabolas) in standard form $f(x)=ax^2+bx+c$
- Graphing quadratic functions (parabolas) in vertex form $f(x)=a(x-h)^2+k$
- Graphing quadratic functions (parabolas) in factored form or x-intercept form
- Writing equations of quadratic functions in vertex form $f(x)=a(x-h)^2+k$
- Writing equations of quadratic functions in factored form $f(x)=a(x-r_1)(x-r_2)$
- Converting a quadratic function from one form to another
- Determining the number of x-intercepts from a vertex form equation $f(x)=a(x-h)^2+k$
- Comparing parabolas based on "a", "h", and "k" in vertex form $f(x)=a(x-h)^2+k$
- Writing an equation of a parabola based on a graph
- Finding the zeros of a quadratic function, i.e, solving $0=a(x-h)^2+k$ or $0=ax^2+bx+c$ or $0=(x-r_1)(x-r_2)$
- The coordinates of x-intercepts
- The coordinates of a y-intercept
- The minimum or maximum value of a quadratic function (the y-value at the vertex)
- The symmetry of a parabola, the axis (line) of symmetry and its equation $x=\dfrac{-b}{2a}$
- Twin points as equidistant to the axis of symmetry
- The vertex and its coordinates
- The concavity of a parabola

Graphing Quadratic Functions in Standard Form $f(x)=ax^2+bx+c$

1. Graph the parabola with quadratic equation $f(x)=x^2$

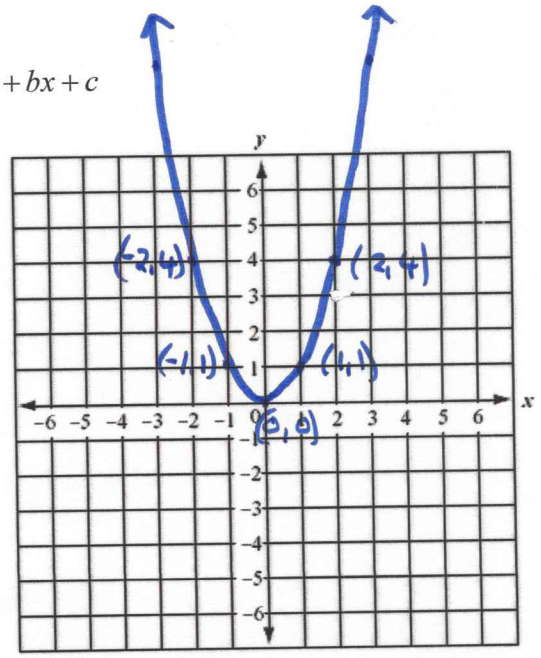
axis $x=\dfrac{-b}{2a}$ $\quad a=1$

$x=\dfrac{-0}{2(1)}=0$ $\quad b=0$

$c=0$

$f(0)=0$ Vertex $V(0,0)$

Pattern: 1, 3, 5, 7, ...



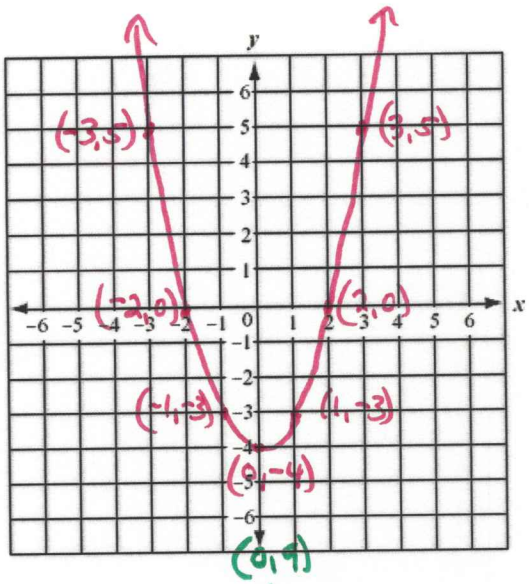
2. Graph the parabola with quadratic equation $f(x) = x^2 - 4$

$f(x) = ax^2 + bx + c \quad a=1 \quad b=0 \quad c=-4$

axis $x = \frac{-b}{2a} = \frac{-0}{2(1)} = 0 \quad x=0$

$f(0) = -4$ Vertex $V(0,-4)$

Pattern: 1, 3, 5, 7, ...



3. Graph the parabola with quadratic equation $f(x) = 9 - x^2$

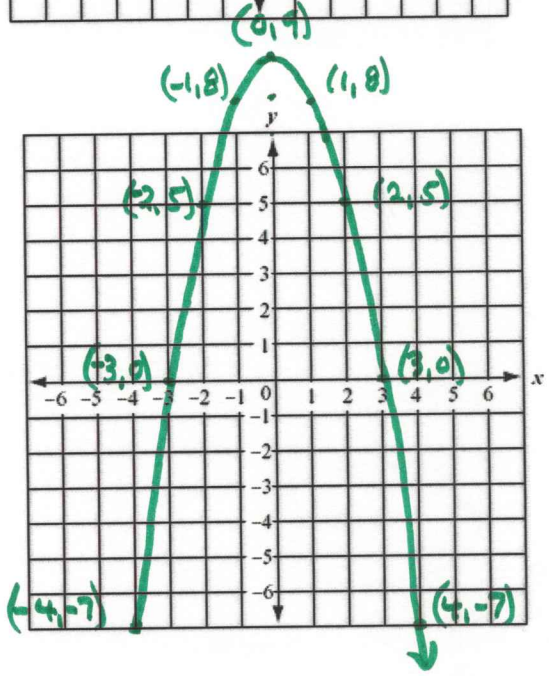
$a=-1 \quad b=0 \quad c=9$

axis $x = \frac{-0}{2(-1)} = 0 \quad x=0$

$f(0) = 9$ Vertex $V(0,9)$

Pattern: 1, 3, 5, 7, ...

$a=-1$ opens downward



4. Graph the parabola with quadratic equation $f(x) = x^2 - 4x$

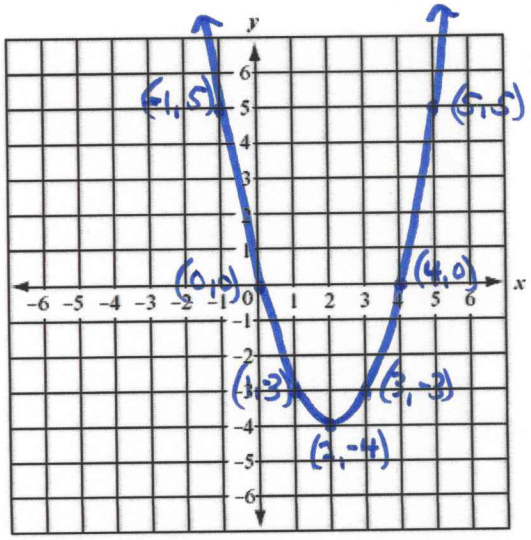
$a=1 \quad b=-4 \quad c=0$

axis $x = \frac{-(-4)}{2(1)} = \frac{4}{2} = 2$

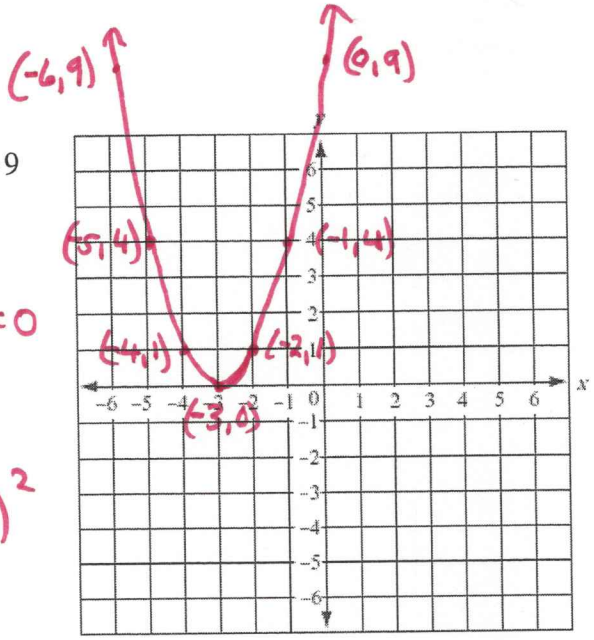
$f(2) = (2)^2 - 4(2) = 4 - 8 = -4$

Vertex $V(2,-4)$

Pattern: 1, 3, 5, 7, ...



5. Graph the parabola with quadratic equation $f(x) = x^2 + 6x + 9$



axis $x = \frac{-b}{2a} = \frac{-6}{2(1)} = -3$ $x = -3$

$f(-3) = (-3)^2 + 6(-3) + 9 = 9 - 18 + 9 = 0$

Vertex $V(-3, 0)$

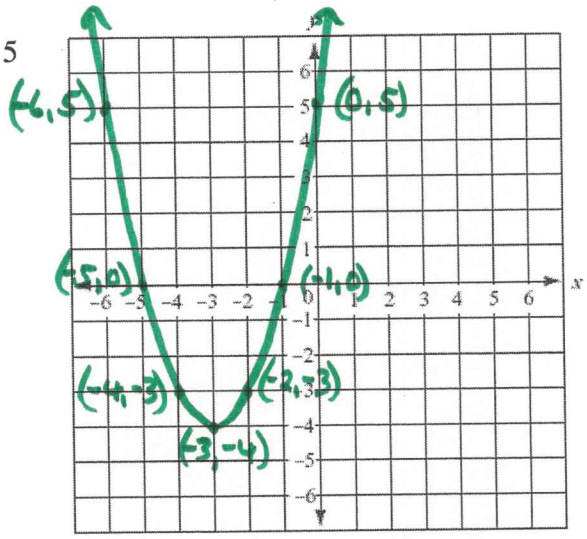
Also $f(x) = (x+3)(x+3) = (x+3)^2$

$y = 0$ $x = -3$ x-intercept $(-3, 0)$

$x = 0$ $y = 9$ y-intercept $(0, 9)$

Pattern: 1, 3, 5, 7, ..

6. Graph the parabola with quadratic equation $f(x) = x^2 + 6x + 5$



axis $x = \frac{-6}{2(1)} = -3$ $x = -3$

$f(-3) = (-3)^2 + 6(-3) + 5 = -4$

Vertex $V(-3, -4)$

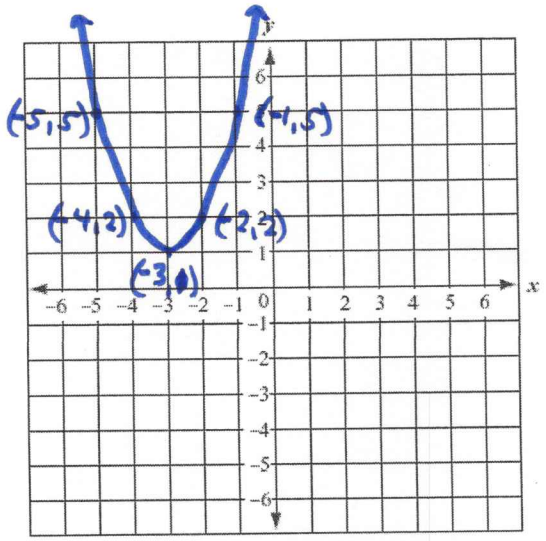
Also $f(x) = (x+1)(x+5)$

x-intercepts $(-1, 0)$ & $(-5, 0)$

$x = 0$ $y = 5$ y-intercept $(0, 5)$

Pattern: 1, 3, 5, 7, ...

7. Graph the parabola with quadratic equation $f(x) = x^2 + 6x + 10$



axis $x = \frac{-6}{2(1)} = \frac{-6}{2} = -3$ $x = -3$

$f(-3) = (-3)^2 + 6(-3) + 10 = 9 - 18 + 10 = 1$

Vertex $V(-3, 1)$

Pattern: 1, 3, 5, 7, ..

$f(0) = -8 \quad f(1) = 2(1)^2 - 6(1) - 8 = -12$

$a = 2$

8. Graph the parabola with quadratic equation $f(x) = 2x^2 - 6x - 8$

$f(x) = 2(x^2 - 3x - 4) = 2(x-4)(x+1)$

x-intercepts $(4,0)$ & $(-1,0)$

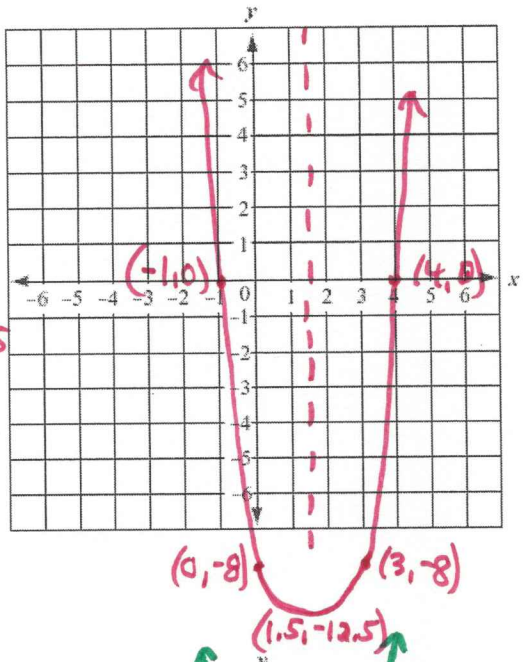
axis of symmetry is at midpoint of x-intercepts so $x = \frac{4+-1}{2} = \frac{3}{2} = 1.5$

$f(1.5) = 2(1.5-4)(1.5+1) = 2(-2.5)(2.5)$

$= -12.5$

$V(1.5, -12.5)$

Pattern: 2, 6, 10, ...



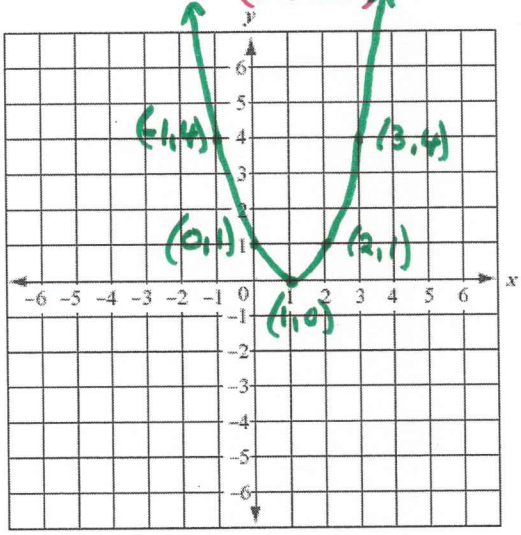
Graphing Quadratic Functions in Vertex Form

$a = 1$

9. Graph the parabola with quadratic equation $f(x) = (x-1)^2$

$f(x) = (x-1)^2 + 0 \quad V(1,0)$

Pattern: 1, 3, 5, 7, ...

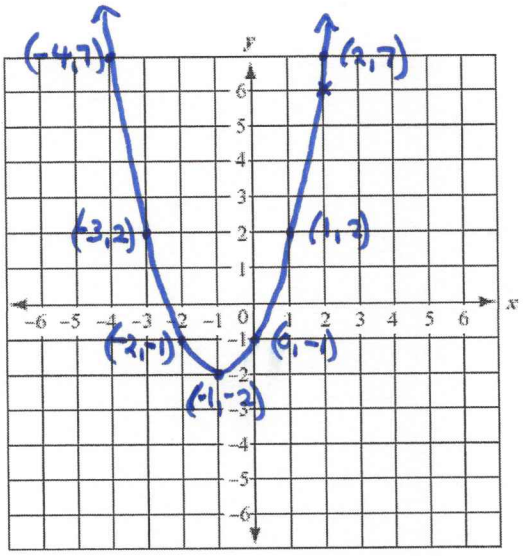


$a = 1$

10. Graph the parabola with quadratic equation $f(x) = (x+1)^2 - 2$

$V(-1,-2)$

Pattern: 1, 3, 5, 7, ...



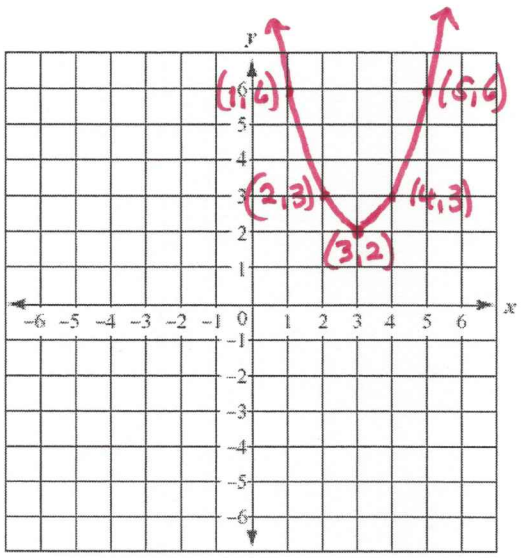
11. Graph the parabola with quadratic equation $f(x)=(x-3)^2+2$

$a=1$

V(3, 2)

no-x-intercepts

Pattern: 1, 3, 5, 7, ..

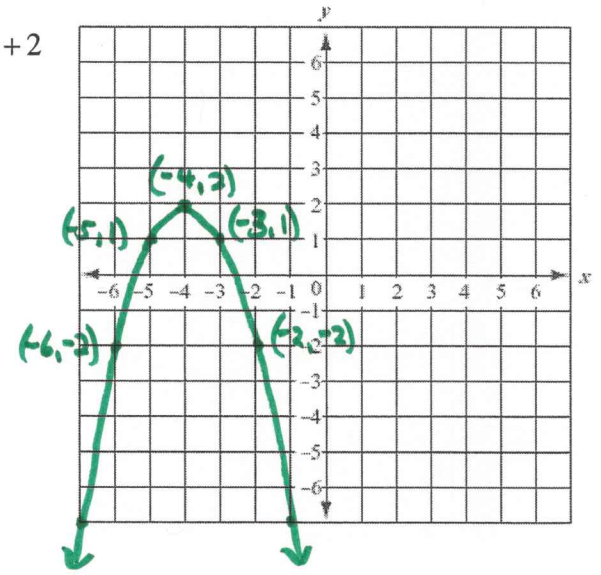


12. Graph the parabola with quadratic equation $f(x)=-(x+4)^2+2$

opens downward $a=-1$

V(-4, 2)

Pattern: 1, 3, 5, 7, ...

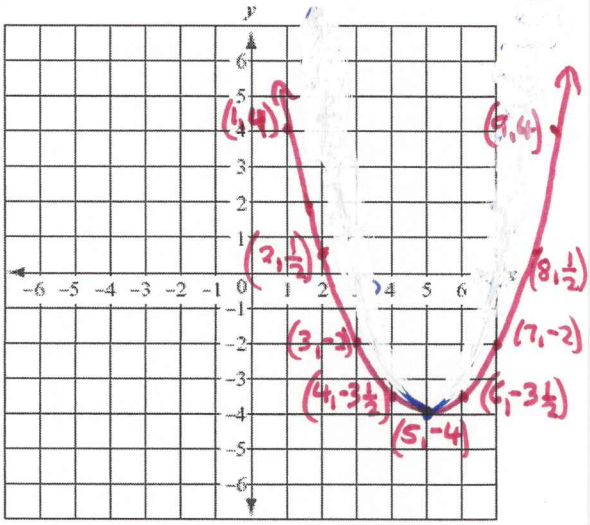


13. Graph the parabola with quadratic equation $f(x)=\frac{1}{2}(x-5)^2-4$

V(5, -4)

$a=\frac{1}{2}$

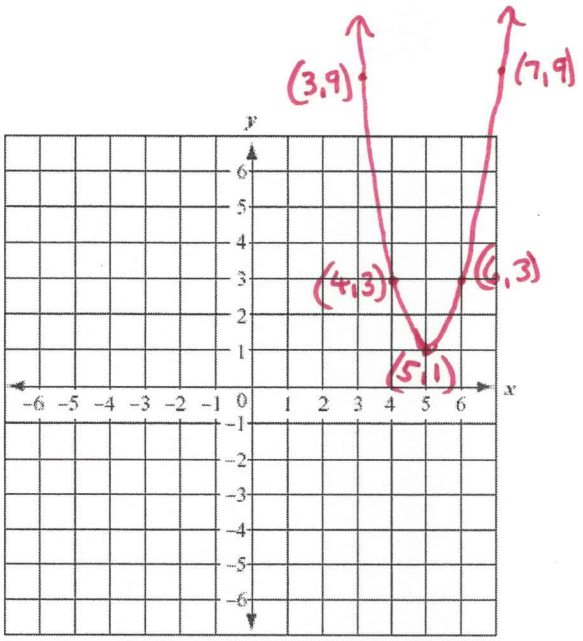
Pattern: $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, ...



14. Graph the parabola with quadratic equation $f(x)=2(x-5)^2+1$

$V(5,1)$ $\quad a=2$

Pattern: $2, 6, 10, \ldots$



Graphing Quadratic Functions in Factored Form or X-intercept form $f(x)=a(x-r_1)(x-r_2)$

15. Graph the parabola with quadratic equation $f(x)=(x-2)(x+2)$

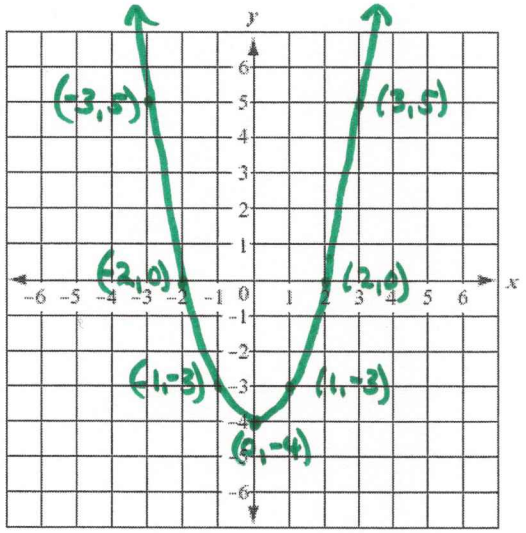
x-intercepts $(2,0)$ & $(-2,0)$

axis $x=0$

$f(0)=(0-2)(0+2)=(-2)(2)=-4$

$V(0,-4)$

Pattern: $1, 3, 5, 7, \ldots$



16. Graph the parabola with quadratic equation $f(x)=(x-5)(x-1)$

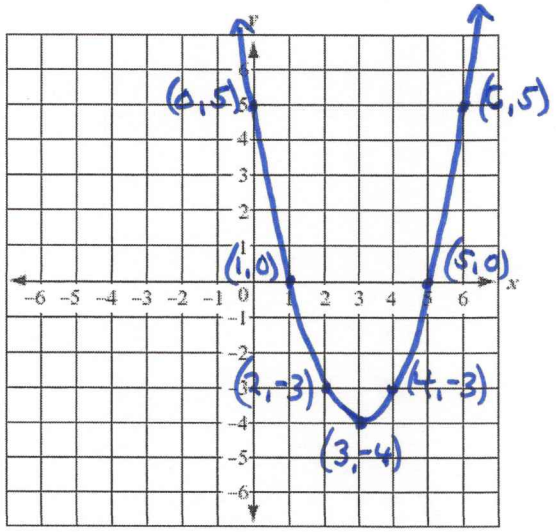
x-intercepts $(5,0)$ & $(1,0)$

axis $x=\frac{1+5}{2}=\frac{6}{2}=3$

$f(3)=(3-5)(3-1)=(-2)(2)=-4$

Vertex $V(3,-4)$

Pattern: $1, 3, 5, 7, \ldots$



17. Graph the parabola with quadratic equation $f(x) = -(x-2)(x+4)$

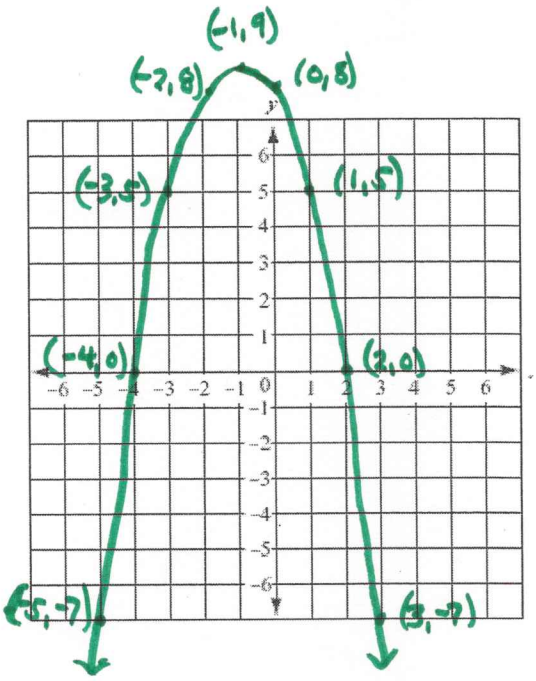
x-intercepts (2,0) & (-4,0)

axis $x = \frac{2+-4}{2} = \frac{-2}{2} = -1$

$f(-1) = -(-1-2)(-1+4) = -(-3)(3) = 9$

Vertex (-1, 9)

Pattern: 1, 3, 5, 7, ...



Writing Equations of Quadratic Functions in Vertex Form $f(x) = a(x-h)^2 + k$

18. Write a vertex-form equation of the parabola with vertex $V(-6,3)$ and point $P(2,35)$

$y = a(x--6)^2 + 3$

$y = a(x+6)^2 + 3$

(2, 35) $35 = a(2+6)^2 + 3$

$32 = a(64)$ $a = \frac{32}{64}$ $a = \frac{1}{2}$

$f(x) = \frac{1}{2}(x+6)^2 + 3$

19. Write a vertex-form equation of the parabola with vertex $V(1,13)$ and point $P(-3,-19)$

V(1, 13) $y = a(x-h)^2 + k$

V(h, k) $y = a(x-1)^2 + 13$

$-19 = a(-3-1)^2 + 13$

$-32 = a(-4)^2$

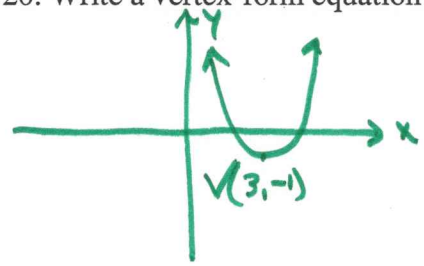
$-32 = a(16)$

$\frac{-32}{16} = a$

$-2 = a$

$y = -2(x-1)^2 + 13$

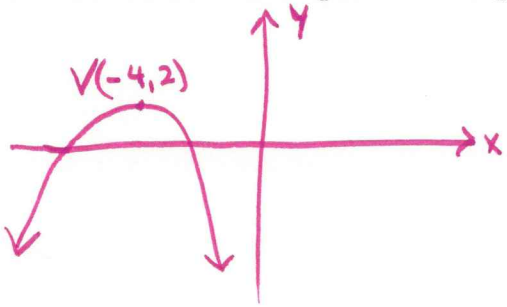
20. Write a vertex-form equation of the parabola that is concave up and has 2 x-intercepts



$y = \frac{1}{2}(x-3)^2 - 1$

one answer of many

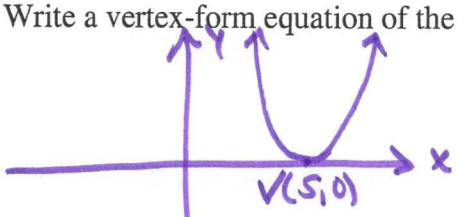
21. Write a vertex-form equation of the parabola that is concave down and has 2 x-intercepts



$y = -3(x+4)^2 + 2$

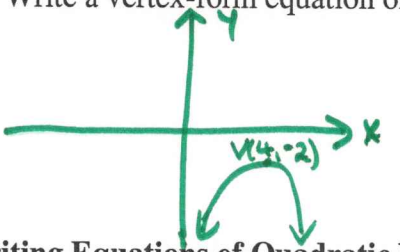
one answer of many

22. Write a vertex-form equation of the parabola that is concave up and has 1 x-intercept



$y=(x-5)^2+0$ one answer of many

23. Write a vertex-form equation of the parabola that is concave down and has no x-intercepts



$y=-(x-4)^2-2$ one answer of many

Writing Equations of Quadratic Functions in Factored Form or X-intercept form

$f(x)=a(x-r_1)(x-r_2)$

24. Write a quadratic equation in factored form given the information. A parabola contains the points $(-2,0)$, $(5,0)$ & $(1,-24)$

x-intercepts (roots) are $x=-2$ & $x=5$

$y=a(x--2)(x-5)$

$y=a(x+2)(x-5)$

use $(1,-24)$

$-24=a(1+2)(1-5)$

$-24=a(3)(-4)$

$-24=a(-12)$

$2=a$

$y=2(x+2)(x-5)$

25. Write a quadratic equation in factored form given the information. A parabola contains the points $(2,0)$, $(-1,0)$ & $(3,2)$

x-intercepts (roots)

$x=2$ & $x=-1$

$y=a(x-2)(x--1)$

$y=a(x-2)(x+1)$

use $(3,2)$

$2=a(3-2)(3+1)$

$2=a(1)(4)$

$2=a(4)$

$a=\frac{1}{2}$

$y=\frac{1}{2}(x-2)(x+1)$

26. Write a quadratic equation in factored form given the information. A parabola contains the points $(-2,0)$, $(2,0)$ & $(1,9)$

x-intercepts

$x=-2$ & $x=2$

$y=a(x+2)(x-2)$

use $(1,9)$

$9=a(1+2)(1-2)$

$9=a(3)(-1)$

$9=a(-3)$

$-3=a$

$y=-3(x+2)(x-2)$

27. Convert the quadratic equation $f(x)=x^2+6x+10$ to vertex form

$f(x)=x^2+6x+10$

$f(x)=(x^2+6x+9)+10-9$

$f(x)=(x+3)(x+3)+1$

$f(x)=(x+3)^2+1$

Converting a quadratic function from one form to another

28. Convert the quadratic function $f(x)=x^2-4x+7$ to vertex form

$f(x)=(x^2-4x+4)+7-4$

$f(x)=(x-2)(x-2)+3$

$f(x)=(x-2)^2+3$ $\quad V(2,3)$

29. Convert the quadratic function $f(x)=2x^2+8x-3$ to vertex form

$f(x)=2(x^2+4x+4)-3+8$

$f(x)=2(x+2)^2+5$

$V(-2,5)$

30. Convert the quadratic function $f(x)=(x-4)^2-10$ to standard form

$f(x)=(x-4)(x-4)-10$

$f(x)=x^2-4x-4x+16-10$

$f(x)=x^2-8x+6$

31. Convert the quadratic function $f(x)=\frac{1}{2}(x+4)^2+3$ to ~~vertex~~ standard form

$f(x)=\frac{1}{2}(x+4)(x+4)+3$

$f(x)=\frac{1}{2}(x^2+8x+16)+3$

$f(x)=\frac{1}{2}x^2+4x+8+3$

$f(x)=\frac{1}{2}x^2+4x+11$

32. Convert the quadratic function $f(x)=(x+1)^2-9$ to factored form

$f(x)=(x+1)(x+1)-9$

$f(x)=x^2+x+x+1-9$

$f(x)=x^2+2x-8$

$f(x)=(x+4)(x-2)$

33. Convert the quadratic function $f(x) = 2x^2 + 5x - 12$ to factored form

$f(x) = (2x - 3)(x + 4)$

Determining the Number of X-intercepts from a Vertex Form Equation

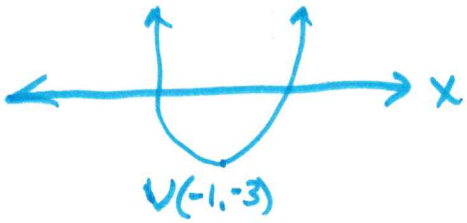
34. Without graphing, determine the number of x-intercepts of the parabola with equation

$f(x) = 3(x-1)^2 + 2$

there are
NO x-intercepts

35. Without graphing, determine the number of x-intercepts of the parabola with equation

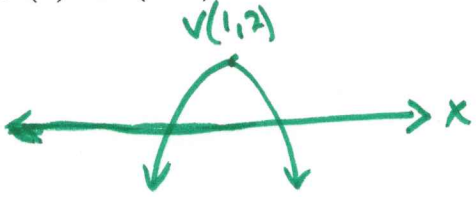
$f(x) = \frac{1}{4}(x+1)^2 - 3$



there are
Two x-intercepts

36. Without graphing, determine the number of x-intercepts of the parabola with equation

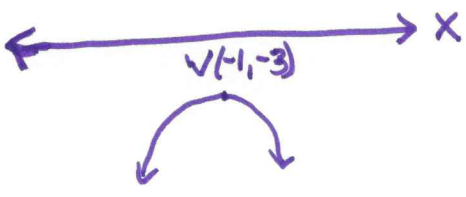
$f(x) = -3(x-1)^2 + 2$



There are
Two x-intercepts

37. Without graphing, determine the number of x-intercepts of the parabola with equation

$f(x) = \frac{-1}{4}(x+1)^2 - 3$



there are
No x-intercepts

Comparing Parabolas Based on "a", "h" and "k" in $f(x)=a(x-h)^2+k$

38. How does the graph of $f(x)=2(x+5)^2+4$ compare to the graph of $f(x)=3(x+1)^2-6$ based on a comparison of the "a" values? Vertex form is $f(x)=a(x-h)^2+k$

Both parabolas open up.

$f(x)=2(x+5)^2+4$ has a parabola that is wider or less steep than $f(x)=3(x+1)^2-6$

$f(x)=2(x+5)^2+4$ has the pattern 2, 6, 10, ...

$f(x)=3(x+1)^2-6$ has the pattern 3, 9, 15, ...

39. How does the graph of $f(x)=2(x+5)^2+4$ compare to the graph of $f(x)=3(x+1)^2-6$ based on a comparison of the "h" values? Vertex form is $f(x)=a(x-h)^2+k$

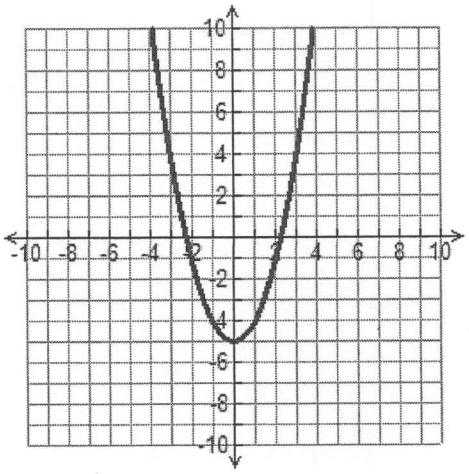
The vertex of $f(x)=2(x+5)^2+4$ is $V(-5,4)$ where as the vertex of $f(x)=3(x+1)^2-6$ is $V(-1,-6)$

40. How does the graph of $f(x)=2(x+5)^2+4$ compare to the graph of $f(x)=3(x+1)^2-6$ based on a comparison of the "k" values? Vertex form is $f(x)=a(x-h)^2+k$

The vertex of $f(x)=2(x+5)^2+4$ is $V(-5,4)$ where as the vertex of $f(x)=3(x+1)^2-6$ is $V(-1,-6)$

Writing an Equation of a Parabola Based on a Graph

41. For the parabola shown, write a quadratic equation in vertex form.

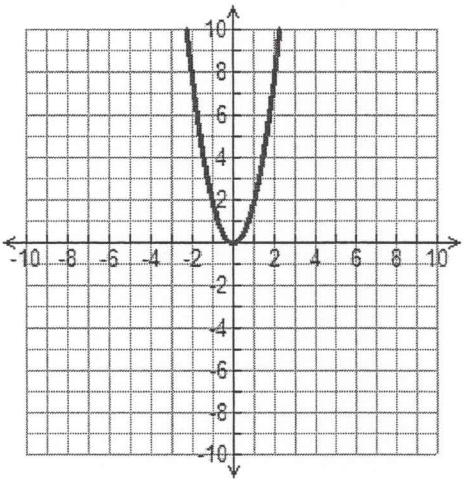


$V(0,-5)$ $f(x)=1(x-0)^2-5$

$f(x)=(x)^2-5$

$f(x)=x^2-5$

42. For the parabola shown, write a quadratic equation in vertex form.

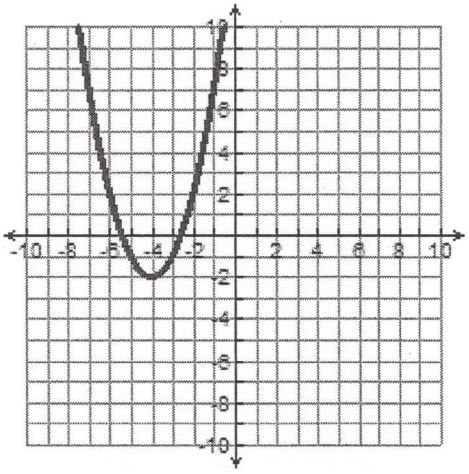


$V(0,0)$ $a=2$

$f(x) = 2(x-0)^2+0$

$f(x) = 2x^2$

43. For the parabola shown, write a quadratic equation in vertex form.

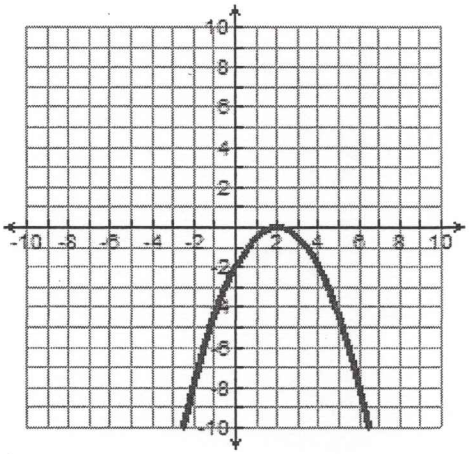


$V(-4,-2)$ $a=1$

$f(x) = (x--4)^2-2$

$f(x) = (x+4)^2-2$

44. For the parabola shown, write a quadratic equation in vertex form.

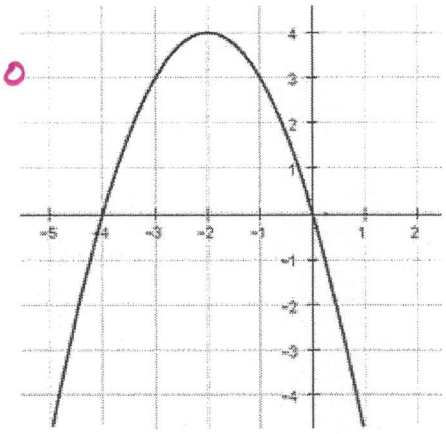


$V(2,0)$ $a=-\frac{1}{2}$

$f(x) = -\frac{1}{2}(x-2)^2+0$

$f(x) = -\frac{1}{2}(x-2)^2$

45. For the parabola shown, write a quadratic equation in factored form.



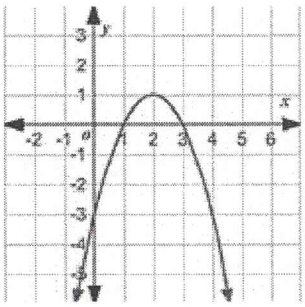
V(-2, 4) x-intercepts x = -4 & x = 0

a = 1 $f(x) = -1(x - -4)(x - 0)$

$f(x) = -(x+4)(x)$

$f(x) = -x(x+4)$

46. For the parabola shown, write a quadratic equation in factored form.



a = -1 x-intercepts x = 1 & x = 3

$f(x) = -1(x-1)(x-3)$

$f(x) = -(x-1)(x-3)$

Finding the zeros of a quadratic function, i.e. solving $0 = a(x-h)^2 + k$ or $0 = ax^2 + bx + c$ or $0 = (x - r_1)(x - r_2)$

47. Determine the zeros of the quadratic function $f(x) = (x-6)(x+1)$, i.e solve $0 = (x-6)(x+1)$

The zeros are the roots or the x-intercepts which is when f(x) = 0 or y = 0.

Solve $0 = (x-6)(x+1)$

$x - 6 = 0$ $x + 1 = 0$

$x = 6$ $x = -1$

zeros

$x = 6$ or $x = -1$

48. Determine the zeros of the quadratic function $f(x) = (2x-3)(x+5)$, i.e solve $0 = (2x-3)(x+5)$

Solve $0 = (2x-3)(x+5)$

$2x - 3 = 0$ or $x + 5 = 0$

$2x = 3$ or $x = -5$

$x = \frac{3}{2}$

Zeros

$x = \frac{3}{2}$ or $x = -5$

49. Determine the zeros of the quadratic function $f(x)=2(x+1)^2-8$, i.e solve $0=2(x+1)^2-8$

Solve $0=2(x+1)^2-8$

$8=2(x+1)^2$

$4=(x+1)^2$

$\pm\sqrt{4}=\sqrt{(x+1)^2}$

$\pm 2 = x+1$

$-1\pm 2 = x$

$x=-1+2$ or $x=-1-2$

$x=1$ or $x=-3$

Zeros

$x=1$ or $x=-3$

50. Determine the zeros of the quadratic function $f(x)=3(x-2)^2+12$, i.e solve $0=3(x-2)^2+12$

Solve $0=3(x-2)^2+12$

$-12=3(x-2)^2$

$-4=(x-2)^2$

No real zeros

51. Determine the zeros of the quadratic function $f(x)=x^2+3x$, i.e solve $0=x^2+3x$

Solve $0=x^2+3x$

$0=x(x+3)$

$x=0$ or $x+3=0$

$x=-3$

Zeros

$x=0$ or $x=-3$

52. Determine the zeros of the quadratic function $f(x)=x^2+10x+24$, i.e solve $0=x^2+10x+24$

Let $f(x)=0$

Solve

$0=x^2+10x+24$

$0=(x+4)(x+6)$

$x+4=0$ $\quad x+6=0$

$x=-4$ $\quad x=-6$

Zeros

$x=-4$ or $x=-6$

53. Determine the zeros of the quadratic function $f(x)=2x^2-9x-18$, i.e solve $0=2x^2-9x-18$

Solve $0=2x^2-9x-18$

$0=(2x+3)(x-6)$

$2x+3=0$ $\quad x-6=0$

$2x=-3$ $\quad x=6$

$x=-\frac{3}{2}$

Zeros

$x=-\frac{3}{2}$ or $x=6$